Auctions

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Auctions...

... are everywhere:

- not only E-bay and Allegro!
- many companies have recently started to use auctions in a day-to-day business
- it is a relatively efficient allocation procedure
- and as such is being applied to an increasing number of areas...

Auctions are considered to be a triumph of game theory.
Part I: General Setting and Definitions
General Settings

- Auctions can be used to allocate goods, tasks, resources, etc.
- Participants:
  - Auctions: auctioneers who sell and (usually many) bidders who buy
  - Reverse Auctions: auctioneer who buys and (usually many) bidders who sell (most common example: procurement auctions)
- Participants agree to rules of the auction and there is some enforcement mechanism
- Why is enforcement needed?
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  - another issue: we adapt our utility - of we bid most than there is a natural human tendency to second-guess decisions made.
The 2007 Nobel Memorial Prize in Economic Sciences was awarded to Leonid Hurwicz, Eric Maskin, and Roger Myerson "for having laid the foundations of mechanism design theory"

- Assumption: private information setting;
- Game designer is interested in the outcome of the game to be designed;
- Say, we design an auction. Of course, every buyer would like to pay less. So, every buyer is interested to pretend that he does not want the good as much as it is really the case;
- How to make agents to disclose their preferences truthfully?
Designing an Auction

1. Who are sellers, who are buyers, what are the items being sold

2. Bidding Rules:
   - sequence of bids (e.g. simultaneous or sequential)
   - units of bids;
   - bidding language.

3. Payment Rules - who pays what;


Bidding rules, payment rules, goods allocation rules are called Auction Mechanisms or Protocols.
One way to classify auctions is with respect to number of sellers, buyers and items being auctioned:

- **Simple Auctions** - 1 seller, \(N\) buyers, 1 item being auctioned
- **Combinatorial Auctions** - 1 seller, \(N\) buyers, \(M\) items being auctioned
- **General Auction Mechanisms** - \(K\) sellers, \(N\) buyers, \(M\) items being auctioned
Classifying Simple Auctions

Four general types of simple auctions:

1. English Auction
2. Dutch Auction
3. First-Price Sealed-Bid Auctions
4. Second-Price Sealed-Bid Auctions

Most of the simple auctions used in practice are variations of the above four types.

There are other auctions like All-Pay Auctions, which are, however, less popular.
Types of Agent’s Values

- **Private value**: value of the good depends only on the agent’s own preferences. E.g. cake which is not resold or showed off;

- **Common value**: agent’s value of an item determined entirely by others’ values. E.g. treasury bills;

- **Correlated value**: agent’s value of an item depends partly on its own preferences and partly on others’ values for it: E.g. auctioning a transportation task when bidders can handle it or re-auction it to others;

**Note**: Bidder also has its own valuation of the good!
Part II: Simple Auctions
An auction mechanism is called "English" if it involves an iterative process of increasing the price. In the case of reverse auction it is decreasing the price. In other words, it is the direction unfavorable to the bidders.

- Usually one seller, multiple bidders, one item;
- **Protocol**: auctioneer starts the first round by announcing a minimum price (called a *reservation price* or Suggested Opening Bid). Buyers can offer higher bids each time starting a new round which lasts only a specified time. If no higher bid is offered within this time frame the auction ends and the highest bidder gets the item at the price of his bid (first-price English auction). Bids are public.
- **Strategy**: depends on buyer’s valuations, his expectations about other buyer’s valuations built partially on the bid history
Dominant Strategy: a bidder should increase his bid a little more every round till either:
- the bidder wins; OR
- he reaches his own valuation of the good.

Since there is dominant strategy no effort is wasted for counterspeculation.

The bidding process can last long.

Applications: antiques, collectors items.

Many variations:
- Reservation price can be hidden and bidding starts from zero
- Second-price English Auction - the winner pays the second highest bid.

English auction is subject to so-called Winner’s Course - it happens when bidders are not certain about their own valuations (discussed later).

During the auction information about true valuations is revealed to others.
An auction mechanism is called "Dutch" if it involves an iterative process of decreasing the price from some arbitrary maximum. In the case of reverse auction it is increasing the price. In other words, it is the direction favorable to the bidders.

- Usually one seller, multiple bidders, one item;
- **Protocol**: the auctioneer starts the first round by announcing a maximum price. Every next round the auctioneer decreases the price a little bit. The first person who accepts wins at the price of the bid (first-price Dutch auction). Bids are public.
- Strategy: wait for a little bit after your true valuation has been called and hope no one else gets in there before you
- **Dominant Strategy**: no general dominant strategy
- During the auction no information is revealed to other buyers and the auctioneer
- **Applications**: flower wholesale markets in Amsterdam, Ontario tobacco auction, Filene’s basement, Waldenbooks
First-Price Sealed-Bid Auction

- Usually one seller, multiple bidders, one item;
- **Protocol**: There is only one round during which bidders submit their bids in closed envelopes. The highest bid wins.
- **Strategy**: bid a little less than your true valuation, i.e. a bid is a function of a private valuation and prior estimates of others’ valuations
- **Dominant Strategy**: ?
- **Applications**: public contracts, public offers (flats), *etc.*
- **Variations**:
  - What if two bidders offered the same price?
  - During the auction information about true valuations is revealed only to the auctioneer

Notes: First-price sealed-bid auction is strategically equivalent to the Dutch auction protocol:

- the buyer with the highest bid gets the good and only his bid matters
- during the auction no information about other valuation gets revealed (to the buyers)
Vickrey Auction = Second-Price Sealed Bid

- Proposed by William Vickrey in 1961
- for his input to the theory of auctions Vickrey was given a Noble Price in 1996 in economics
- Protocol: There is only one round during which bidders submit their bids in closed envelopes. The bidder with the highest bid wins but pays the second highest price.
- Dominant strategy: bid the true valuation
  - if you bid more, you risk to pay too much
  - if you bid less, you lower your chances of winning while still having to pay the same price in case you do win
- No effort is wasted for counterspeculation
- Problem: counter-intuitive — it is difficult to explain it to people
- For private value auctions, Vickrey auction becomes strategically equivalent to the English auction mechanism (but not if there are big obligatory increments between bids).
The Winner’s Course may occur in an auction when bidders being uncertain of true value of auctioned goods. If they estimate the value wrongly then the winner will overpay. A well-known example are auctions of oil fields. This version of this example is due to Vincent Conitzer, Duke University:

- Bidding on drilling rights for an oil field.
- Each bidder $i$ has its own geologists who do tests, based on which the bidder assesses an expected value $v_i$ of the field
- If you win, it is probably because the other bidders’ geologists’ tests turned out worse, and the oil field is not actually worth as much as you thought
- Bidding $v_i$ is no longer a dominant strategy in the second-price auction
Comparison of Simple Auctions

- All four auctions allocate item efficiently if we assume no reservation price for the auctioneer;
- Dutch auction is strategically equivalent to first-price sealed-bid auction;
- Vickrey auction is strategically equivalent to English auction if agents are risk neutral;
- English and Vickrey auctions have dominant strategies; hence no effort is wasted for counterspeculation;
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Show me the money moment...
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Show me the money moment...

Which of the four auction mechanisms gives highest expected revenue to the seller?
Revenue Equivalence Theorem

Given by Vickrey, 1961

Assuming:
- bidders are risk-neutral; and
- their valuations are drawn from a uniform distribution

then English, Dutch, FPSB and Vickrey auctions result in the same expected revenue of the auctioneer. This revenue \( \approx \) second highest valuation.

Intuition behind the proof:
- English auction — bidding stops just after second highest valuation;
- Dutch/FPSB auctions — bid \( \approx \) second highest valuation due to the uniform value distribution;
- Vickrey auction — as already discussed.
Let us waive the assumption of risk-neutrality of bidders and auctioneer. In such a case, revenue equivalence ceases to hold:

- If bidders are risk averse — Dutch and FPSB auctions have better expected revenue than Vickrey and English auctions;
- If auctioneer is risk averse — Dutch and FPSB auctions have worse expected revenue than Vickrey and English auctions;
All-Pay Auction

- Usually one seller, multiple bidders, one item;

- **Protocol:** Bidders are free to rise their bids. The auction ends when nobody wants to rise his bid. The highest bidder is the winner and he received the item. However, all the bidders are supposed to pay their last bids to the auctioneer.

- **Strategy:** depends on buyer’s valuations, his expectations about other buyer’s valuations built partially on the bid history

- **Dominant Strategy:** see Wikipedia on "Bidding fee scheme" term. They offer an interesting overview. In general, try to find websites that have just started and competition is relatively small. On the other hand, with new companies there is a risk of being cheated.

- **Applications:** tool reallocation, Lenting and Braspennning ECAI, 1994.

- **Variations:**
  - Each buyer is obliged to pay only a part of his highest bid
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Vulnerability of Simple Auctions

- Players collusion
  - \( v_1 = 50 \) and all the others have \( v_{i\neq 1} = 40 \)
  - English auction - any \( b_1 > b_i > 0 \)
  - Vickrey auction - any \( b_1 > b_i > 0 \)
  - FPSB and Dutch auctions - if \( b_1 < 40 \) others have incentive to break the collusion agreement.

- Dishonest auctioneer — only in auctions in which bids are not public